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A Rudimentary Operations on Octagonal Fuzzy Numbers

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Abstract- Fuzzy numbers and their fuzzy operations are the basis of fuzzy number theory. Fuzzy arithmetic is a system of fuzzy operations on fuzzy numbers. The operations in the set of fuzzy numbers are developed by the Zadeh extension principle. In this paper, the α-cut of octagonal fuzzy number is defined and rudimentary operations are carried out using interim arithmetic of α-cut and embellished by numerical examples.

Index Terms- Fuzzy number; octagonal fuzzy number; rudimentary operations; α-cut.

1. INTRODUCTION

In 1965, Lotfi Aliaskar Zadeh published his innovating paper "Fuzzy Set" in the journal of Information and Control [1]. Since then, Lotfi Aliaskar Zadeh was consider as the founder and father of fuzzy set theory and fuzzy logic. Fuzzy set has been infiltrating into almost all branches of pure and applied mathematics that are set-theory-based. This has resulted in a vast number of real applications crossing over a broad realm of domains and disciplines.

Over the years, many of the existing approaches dealing with imprecision and uncertainty are based on the theory of fuzzy sets [1] and possibility theory [2]. Fuzzy is a new mathematical tool introduced in 1965 to handle data and information having non-statistical uncertainties. Fuzzy was particularly intended to mathematically represent uncertainty and vagueness. Fuzzy numbers and their fuzzy operations are foundations of fuzzy number theory. Fuzzy numbers are initially employed by Lotfi Aliaskar Zadeh [2]. Fuzzy arithmetic is built on two properties of fuzzy numbers that is each fuzzy set and also each fuzzy numbers can be totally manipulated by its α-cut and α-cut of each fuzzy number are closed intervals of real numbers for all $\alpha \in [0,1]$. There are two methods for expanding fuzzy arithmetic. First method is interim arithmetic and the other method manipulates the extension principle by which operations on real number are extended to operations on fuzzy numbers. In this paper, some operations were done using fuzzy numbers. We also define some of the basic arithmetic operations of octagonal fuzzy number using arithmetic interim of α-cut and is embellished by numerical examples.

2. PRELIMINARIES

In this section, we give the preliminaries that are required for this study.

Definition 2.1. [1] *A fuzzy set A is defined by* $A = \{(x, \mu_A(x)): x \in A, \mu_A(x) \in [0,1]\}$. *Here* x *is crisp set A and* $\mu_A(x)$ *is membership function in the interval* $[0,1]$.

Definition 2.2. [3] *The fuzzy number A is a fuzzy set whose membership function must satisfy the following conditions.*
(i) A fuzzy set A of the universe of discourse X is convex
(ii) A fuzzy set A of the universe of discourse X is a normal fuzzy set if $x_i \in X$ exists
(iii) $\mu_A(x)$ is piecewise continuous

Definition 2.3. [4] *A fuzzy number* $A = (a, b, c)$, *where* $a \leq b \leq c$, *is triangular fuzzy number and its membership function is given by*

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, for a \leq x \leq b \\ \frac{c-x}{c-b}, for b \leq x \leq c \\ 0, x > c \end{cases}$$

Definition 2.4 [5] *A fuzzy number* $A = (a, b, c, d)$, *where* $a \leq b \leq c \leq d$, *is trapezoidal fuzzy number and its membership function is given by*

$$\mu_A(x) = \begin{cases} 0, forx < a \\ \frac{x-a}{b-a}, fora \le x \le b \\ 1, forb \le x \le c \\ \frac{d-x}{d-c}, forc \le x \le d \\ 0, x > d \end{cases}$$

Definition 2.5. *An α-cut of fuzzy set A is classical set defined as* ${}^{\alpha}[A] = \{x \in X | \mu_A(x) \ge \alpha\}$

Definition 2.6. *A fuzzy set A is a convex fuzzy set iff each of its α-cut ${}^{\alpha}A$ is a convex set.*

3. OCTAGONAL FUZZY NUMBER

A fuzzy number $A_{oct} = (a, b, c, d, e, f, g, h)$, where $a \le b \le c \le d \le e \le f \le g \le h$, is called an octagonal fuzzy number and its membership function is given by (where $0 < k < 1$)

$$\mu_A(x) = \begin{cases} 0, forx < a \\ k\left(\frac{x-a}{b-a}\right), for \ a \le x \le b \\ k, forb \le x \le c \\ k + (1-k)\left(\frac{x-c}{d-c}\right), for \ c \le x \le d \\ 1, ford \le x \le e \\ k + (1-k)\left(\frac{f-x}{f-e}\right), for \ e \le x \le f \\ k, for \ f \le x \le g \\ k\left(\frac{h-x}{h-g}\right), for \ g \le x \le h \\ 0, for \ x > h \end{cases}$$

3.1. *Rudimentary operations on octagonal fuzzy number*

3.1.1. *Addition of two octagonal fuzzy number*

Let $A = (a, b, c, d, e, f, g, h)$ and $B = (a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1)$ be two octagonal fuzzy numbers, then $A + B = [(a + a_1), (b + b_1), (c + c_1), (d + d_1), (e + e_1), (f + f_1), (g + g_1), (h + h_1)]$

Example 3.1.1. :

Let $A = (1,2,3,5,6,8,9,11)$ and $B = (1,3,4,5,6,7,8,10)$ then $A + B = [2,5,7,10,12,15,17,21]$

3.1.2. *Subtraction of two octagonal fuzzy number*

Let $A = (a, b, c, d, e, f, g, h)$ and $B = (a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1)$ be two octagonal fuzzy numbers, then $A - B = [(a - a_1), (b - b_1), (c - c_1), (d - d_1), (e - e_1), (f - f_1), (g - g_1), (h - h_1)]$

Example 3.1.2.:

Let $A = (1,3,7,9,11,12,13,14)$ and $B = (0,1,2,3,4,7,8,10)$ then $A - B = [1,2,5,6,7,5,5,4]$

3.1.3. *Multiplication of two octagonal fuzzy number*

Let $A = (a, b, c, d, e, f, g, h)$ and $B = (a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1)$ be two octagonal fuzzy numbers, then $A * B = [(a * a_1), (b * b_1), (c * c_1), (d * d_1), (e * e_1), (f * f_1), (g * g_1), (h * h_1)]$

Example 3.1.3.:

Let $A = (0,1,2,3,4,5,6,7)$ and $B = (1,2,3,4,5,6,7,8)$ then $A * B = [0,2,6,12,20,30,42,56]$

4. ALPHA CUT

Definition 4.1. *For* $\alpha \in [0,1]$, *the α-cut of an octagonal fuzzy number* $A = (a, b, c, d, e, f, g, h)$ *is defined as*

$${}^{\alpha}A_{oct} = \begin{cases} [a + (\frac{\alpha}{k})(b-a), h - (\frac{\alpha}{k})(h-g)], \\ for\alpha \in [0, k] \\ [c + (\frac{\alpha - k}{1-k})(d-c), f - (\frac{\alpha - k}{1-k})(f-e), \\ for\alpha \in [k, 1] \end{cases}$$

4.1. *Operations of octagonal fuzzy number using α-cut*

The α-cut of octagonal fuzzy number $A = (a, b, c, d, e, f, g, h)$ and $B = (a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1)$ for all $\alpha \in [0,1]$ when $k = 1/2$ is given by

$${}^{\alpha}A_{oct} = \begin{cases} [a + 2\alpha(b-a), h - 2\alpha(h-g)], \\ \quad for \ \ \alpha \in [0,0.5] \\ [c + (2\alpha - 1)(d-c), \\ f - (2\alpha-1)(f-e)], \quad for \ \ \alpha \in [0.5,1] \end{cases}$$

$${}^{\alpha}B_{oct} = \begin{cases} [a_1 + 2\alpha(b_1 - a_1), h_1 - 2\alpha(h_1 - g_1)], \\ \quad for \ \ \alpha \in [0,0.5] \\ [c_1 + (2\alpha - 1)(d_1 - c_1), f_1 - (2\alpha - 1) \\ (f_1 - e_1)], \quad for \ \ \alpha \in [0.5,1] \end{cases}$$

4.1.1. ***Addition of two octagonal fuzzy number***

Let $A = (a,b,c,d,e,f,g,h)$ and $B = (a_1,b_1,c_1,d_1,e_1,f_1,g_1,h_1)$ be two octagonal fuzzy numbers. Let us add the α-cut of ${}^{\alpha}[A_{oct}]+{}^{\alpha}[B_{oct}]$ of A and B using interim arithmetic.

$$ {}^{\alpha}[A_{oct}]+{}^{\alpha}[B_{oct}] = \begin{cases} [a+2\alpha(b-a), h-2\alpha(h-g)] \\ +[a_1+2\alpha(b_1-a_1), \\ h_1-2\alpha(h_1-g_1)], \\ for \quad \alpha \in [0,0.5] \\ [c+(2\alpha-1)(d-c), \\ f-(2\alpha-1)(f-e)] \\ +[c_1+(2\alpha-1)(d_1-c_1), \\ f_1-(2\alpha-1)(f_1-e_1)], \\ for \quad \alpha \in [0.5,1] \end{cases} $$

Example 4.1.1.:

Let $A = (1,2,3,5,6,8,9,11)$ and $B = (1,3,4,5,6,7,8,10)$. For $\alpha \in [0,0.5]$, we ha ${}^{\alpha}[A_{oct}] = [1+2\alpha(2-1), 11-2\alpha(11-9)] = [1+2\alpha, 11-4\alpha]$ and ${}^{\alpha}[B_{oct}] = [1+2\alpha(3-1), 10-2\alpha(10-8)] = [1+4\alpha, 10-4\alpha]$. Therefore, ${}^{\alpha}[A_{oct}]+{}^{\alpha}[B_{oct}] = [2+6\alpha, 21-8\alpha]$. Moreover, when $\alpha = 0$, ${}^{\alpha}[A_{oct}]+{}^{\alpha}[B_{oct}] = [2,21]$ and when $\alpha = 0.5$, ${}^{\alpha}[A_{oct}]+{}^{\alpha}[B_{oct}] = [5,17]$.

For $\alpha \in [0.5,1]$, we have ${}^{\alpha}[A_{oct}] = [3+(2\alpha-1)2, 8-(2\alpha-1)2] = [1+4\alpha, 10-4\alpha]$ and ${}^{\alpha}[B_{oct}] = [4+(2\alpha-1)1, 7-(2\alpha-1)1] = [3+2\alpha, 8-2\alpha]$. Therefore, ${}^{\alpha}[A_{oct}]+{}^{\alpha}[B_{oct}] = [4+6\alpha, 18-6\alpha]$. Moreover, when $\alpha = 0.5$, ${}^{\alpha}[A_{oct}]+{}^{\alpha}[B_{oct}] = [7,15]$ and when $\alpha = 1$, ${}^{\alpha}[A_{oct}]+{}^{\alpha}[B_{oct}] = [10,12]$. Here ${}^{\alpha}[A_{oct}]+{}^{\alpha}[B_{oct}] = [2,5,7,10,12,15,17,21]$.

4.1.2. ***Subtraction of two octagonal fuzzy number***

Let $A = (a,b,c,d,e,f,g,h)$ and $B = (a_1,b_1,c_1,d_1,e_1,f_1,g_1,h_1)$ be two octagonal fuzzy numbers. Let us subtract the α-cut of ${}^{\alpha}[A_{oct}]-{}^{\alpha}[B_{oct}]$ of A and B using interim arithmetic.

$$ {}^{\alpha}[A_{oct}]-{}^{\alpha}[B_{oct}] = \begin{cases} [a+2\alpha(b-a), h-2\alpha(h-g)] \\ -[a_1+2\alpha(b_1-a_1), \\ h_1-2\alpha(h_1-g_1)], \\ \quad for \quad \alpha \in [0,0.5] \\ [c+(2\alpha-1)(d-c), \\ f-(2\alpha-1)(f-e)] \\ -[c_1+(2\alpha-1)(d_1-c_1), \\ f_1-(2\alpha-1)(f_1-e_1)], \\ \quad for \quad \alpha \in [0.5,1] \end{cases} $$

Example 4.1.2. :

Let $A = (1,3,7,9,11,12,13,14)$ and $B = (0,1,2,3,4,7,8,10)$. For $\alpha \in [0,0.5]$, we have ${}^{\alpha}[A_{oct}] = [1+2\alpha(3-1), 14-2\alpha(14-13)] = [1+4\alpha, 14-2\alpha]$ and ${}^{\alpha}[B_{oct}] = [0+2\alpha(1-0), 10-2\alpha(10-8)] = [2\alpha, 10-4\alpha]$. Therefore, ${}^{\alpha}[A_{oct}]-{}^{\alpha}[B_{oct}] = [1+2\alpha, 4+2\alpha]$. Moreover, when $\alpha = 0+6$, ${}^{\alpha}[A_{oct}]-{}^{\alpha}[B_{oct}] = [1,4]$ and when $\alpha = 0.5$, ${}^{\alpha}[A_{oct}]-{}^{\alpha}[B_{oct}] = [2,5]$.

For $\alpha \in [0.5,1]$, we have ${}^{\alpha}[A_{oct}] = [7+(2\alpha-1)2, 12-(2\alpha-1)1] = [5+4\alpha, 13-2\alpha]$ and ${}^{\alpha}[B_{oct}] = [2+(2\alpha-1)1, 7-(2\alpha-1)3] = [1+2\alpha, 10-6\alpha]$. Therefore, ${}^{\alpha}[A_{oct}]-{}^{\alpha}[B_{oct}] = [4+2\alpha, 3+4\alpha]$. Moreover, when $\alpha = 0.5$, ${}^{\alpha}[A_{oct}]-{}^{\alpha}[B_{oct}] = [5,5]$ and when $\alpha = 1$, ${}^{\alpha}[A_{oct}]-{}^{\alpha}[B_{oct}] = [6,7]$. Here ${}^{\alpha}[A_{oct}]-{}^{\alpha}[B_{oct}] = [1,2,5,6,7,5,5,4]$.

4.1.3. ***Multiplication of two octagonal fuzzy number***

Let $A = (a,b,c,d,e,f,g,h)$ and $B = (a_1,b_1,c_1,d_1,e_1,f_1,g_1,h_1)$ be two octagonal fuzzy numbers. Let us multiply the α-cut of ${}^{\alpha}[A_{oct}] *^{\alpha} [B_{oct}]$ of A and B using interim arithmetic.

$$ {}^{\alpha}[A_{oct}] *^{\alpha} [B_{oct}] = \begin{cases} [a+2\alpha(b-a), h-2\alpha(h-g)] \\ *[a_1+2\alpha(b_1-a_1), \\ h_1-2\alpha(h_1-g_1)], \\ \quad for \quad \alpha \in [0,0.5] \\ [c+(2\alpha-1)(d-c), \\ f-(2\alpha-1)(f-e)] \\ *[c_1+(2\alpha-1)(d_1-c_1), \\ f_1-(2\alpha-1)(f_1-e_1)], \\ \quad for \quad \alpha \in [0.5,1] \end{cases} $$

Example 4.1.3. :

Let $A = (0,1,2,3,4,5,6,7)$ and $B = (1,2,3,4,5,6,7,8)$. For $\alpha \in [0,0.5]$, we have ${}^{\alpha}[A_{oct}] = [0+2\alpha(1-0), 7-2\alpha(7-6)] = [2\alpha, 7-2\alpha]$ and ${}^{\alpha}[B_{oct}] = [1+2\alpha(2-1), 8-2\alpha(8-7)] = [1+2\alpha, 8-2\alpha]$. Therefore, ${}^{\alpha}[A_{oct}] *^{\alpha} [B_{oct}] = [4\alpha^2+2\alpha, 56-30\alpha+4\alpha^2]$. Moreover, when $\alpha = 0$, ${}^{\alpha}[A_{oct}] *^{\alpha} [B_{oct}] = [0,56]$ and when $\alpha = 0.5$, ${}^{\alpha}[A_{oct}] *^{\alpha} [B_{oct}] = [2,42]$.

For $\alpha \in [0.5,1]$, we have ${}^{\alpha}[A_{oct}] = [2+(2\alpha-1)(3-2), 5-(2\alpha-1)(5-4)] = [1+2\alpha, 6-2\alpha]$ and ${}^{\alpha}[B_{oct}] = [3+(2\alpha-1)(4-3), 6-(2\alpha-1)(6-5)] = [2+2\alpha, 7-2\alpha]$. Therefore,

${}^{\alpha}[A_{oct}] *^{\alpha} [B_{oct}] = [4\alpha^2 + 6\alpha + 2, 4\alpha^2 - 26\alpha + 42]$. Moreover, when $\alpha = 0.5$, ${}^{\alpha}[A_{oct}] *^{\alpha} [B_{oct}] = [6,30]$ and when $\alpha = 1$, ${}^{\alpha}[A_{oct}] *^{\alpha} [B_{oct}] = [12,20]$. Here ${}^{\alpha}[A_{oct}] *^{\alpha} [B_{oct}] = [0,2,6,12,20,30,42,56]$.

5. CONCLUSION

In this paper, the rudimentary operations are carried out with arithmetic interim of α-cuts and are embellished by numerical examples. Octagonal fuzzy number can be applied to that problem which has eight points in representation. In future it may be applied in optimization technique problems.

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